

## **My mathematical journey**

**by Jim Adams, 11<sup>th</sup> August 2011,**

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Since this personal mathematical history was begun in 2011, much insight has been gained in my mathematical enterprise, so that I am now constructing an overarching description of mathematics – the eBook *Superexponential algebra*, in three volumes, and this interrelates superexponential algebra with its corresponding analogues in geometry and in logic, which are generalisations of the subject as it is most often taught. It is now no longer premature to give a history of the hyperintricate representation of matrices, for which a colleague had suggested I give some historical background of my mathematical journey, so that the early work can be put in context.

I don't have a degree, and my work on mathematics started in earnest in 2008 at the age of 58, after my career as a fairly senior payroll analyst programmer at P&O Properties had finished.

I introduced myself to groups and topology at the age of 15, and to general relativity at the age of 18. I have been a passive reader of advanced texts in mathematics and theoretical physics for a long time.

The situation became more active after I had learnt what it meant to compose music, to have my compositions performed, and even – hoisted by my own petard when I was organising a concert, and the pianist for my work withdrew – playing my own compositions myself to a live audience. This taught me that it was possible to produce music in a style that I wished to present – nudging at boundaries technically, in terms of the sonorities, and in the message that I occasionally wanted to project – yet at the same time I had little or no formal training. In this I was assisted by New Music Brighton, a self help group of composers, but it still took 5 years to get my first composition performed. To a certain extent, I was surprised by my own success. It showed me that it is possible to do things out of the ordinary, and yet be an ordinary person. That is how I developed the courage to embark on mathematical research.

If you look at my website – well some of the early stuff is suitably brushed under the carpet – you will see there is an early work containing over 100 theorems on exponentiation. What I did, was start on something that I knew how to manipulate, and hoped I could produce results in – number theory. This is not very fashionable nowadays – the heyday of number theory was the 19<sup>th</sup> century (or even before). There are no major surprises in this work, except the demonstrated capacity to deal with detail, and to carry an idea to its technical exhaustion.

The first major work was what I thought would be a fairly simple task – solving an unsolved problem in what are called quadratic residues, using elementary methods – that is, not using complex numbers. A quadratic residue is the square of a number in clock arithmetic, in this case 12 o'clock is 0 and there are a prime number of hours. The problem I chose has been solved by other, transcendental methods, but it is stated by Richard Guy as an unsolved

problem in his “Unsolved problems in number theory” book. I thought it would take me a month to solve it, but I had bitten off almost more than I could chew. It is still incomplete, other tasks intervened, and loads of calculation had to be performed for the first stage. Eventually I “cheated”. I looked at Hermann Weyl’s proof of the transcendental result, and adopted part of it, Minkowski’s lattice theory, so that its contents, but not form, were present in my proof. However, when I tried to submit the paper to arXiv, Steven Miller who I was in contact with, seemed dubious about the proof. On trying to tighten it up, once again the proof failed. So the proof of the failure is now in part I of the proof. Part II is quite amazing and beautiful – I have found patterns that I did not expect to find, because I had to look into greater detail because of the failure, and was looking at prime number 1031, and found general reasons why these patterns, or ‘parabolas’ exist. The proof is still not complete, but now has good prospects – I hope! The problem also seems to be related to the ‘tenth discriminant’ problem, which has been solved, but with great difficulty. So what I was doing was try to solve this difficult problem by elementary methods.

I have been interested in my life in a number of mathematical ideas. One that caused me grief at university was the consistency of the real number system. This doesn’t cause me problems nowadays, because I am acquainted with intuitionistic logic, and I am aware there are several components to the axiom system for real numbers, like for example the continuum hypothesis. What I have developed in the work on infinitesimals is the “Archimedean” axiom that any real number can be multiplied by a natural number to be greater than any natural number – this was known to Euclid. The real numbers do not have to conform to this, as was known to Poincaré, but the modern conception excludes these infinitesimals by definition, but there is work by Paul Cohen on the axiom of choice that allows them back in. This work has met quite a lot of resistance, and it has become apparent that to avoid confusion, I have to use another word than real, so I have chosen ladder number. There are two differences to my definition; ladder numbers are not well-ordered, and I am using uncountable induction, a technique that was used by Gentzen in the 1930’s, in circumstances where the theory of real numbers does not allow it; they are constructible. The model I use has multiple versions of sets with internal and external ordering. I have put the work on a basis so that the assumptions are clear, so it should be evident that I am not challenging conventional proofs, but I still might not have acceptance of the axioms I am using. A second work on a similar theme is to allocate zero divided by zero to the *set* of real numbers.

This work, which was described in the eBook *Innovation in mathematics*, has now reached a more mature stage, and is presented in the eBook *Superexponential algebra*. The first new idea is to introduce a modification of set theory I have called mZFC – modified Zermelo-Fraenkel set theory with the axiom of choice. This redefines the empty set as satisfying a false condition. Set theory is not now confined to studying true statements, since if it is false, say for the Russell’s paradox, where we consider a set which does not contain itself, the set is empty. This now means we can use the principle of induction for properties. We use it to show rigourously that the base assumption of the continuum hypothesis is incompatible with the countability of the rational numbers.

Another “problem” for me has been understanding Galois theory. My stance at first was that perhaps I both did and did not accept it. Here is what I said in 2011.

The concrete part of this is that no polynomial of degree greater than 4 is “solvable by radicals”, and what I didn’t see, and originally I didn’t understand the proof, was where the information was being “lost” so that the problem was unsolvable. Because I didn’t understand the proof, I was trying the impossible – to solve the quintic. However, despite (usually) accepting this result, I believed the underlying question was still valid. The reasoning was, if the underlying reason is group theory – which can be reduced to permutations, and if I can generalise numbers so that a permutation can be represented by one of these numbers – this isn’t the case for complex numbers or quaternions – then can I solve the quintic. A more recent realisation was that what I call substitution methods are ineffective in producing solutions, and this is because the group theory is multiplicative, and the multiplicative part is to do with powers of numbers, so I have to factorise 5 into something smaller (as was stated by Galois), and this *is* possible,  $5 = (2 + i)(2 - i)$ , but in fact this doesn’t solve the quintic, we have to go into non-commutative numbers, which I will mention next, so that we can explore maps onto permutations of roots.

Well, I now have something to say on Galois theory, as a vindication of my obdurate intuition, but it still came as a shock to me that the theory is wrong, even though one of its end results holds – no solution of the general quintic or higher degree polynomials by radicals is possible, which can be proved independently of group theory by using the theory of varieties. Basically, a polynomial can be represented in two ways, as the roots  $(x + a)$  etc. multiplied together, or additively as a polynomial. When it is in multiplicative form, we can solve it when it is set to zero. So any theory which uses only multiplication and then states that we cannot get a solution that way has got it wrong. This is Galois theory, and it uses groups. For a polynomial in additive format, we need to go beyond groups to consider rings. Unfortunately, when we employ what are called ring automorphisms, the group theory breaks down – ring automorphisms are involutions (applied twice they get back to the original root) and for complex numbers they are commutative:  $AB = BA$ , unlike the groups of Galois theory. We can confirm what we have said is correct, because if I have, say, the polynomial equation  $(x + a)(x + b)(x + c) = 0$ , then there is no general function just using complex numbers which swaps  $a$  and  $b$  and leaves  $c$  fixed – and the mantra ‘leaving  $c$  fixed’ is used often in Galois theory to describe its solvability results.

Well, apart from work on division algebras, and the work of my namesake, J. F. Adams, on this in the 1960’s, what motivated me to introduce the hyperintricate representation was the search for a number system that was non-commutative, as I thought was necessary to solve “the impossible”, by Galois theory. I wanted something that would incorporate complex numbers at one end, but also be non-commutative. The idea came from R. Remmert in a book called Numbers (by Ebbinghaus et al), although I had looked at the matrix representation of quaternions in a book by Herstein before this. It gave the matrix representation of complex numbers. I immediately realised that an extension of this idea was possible – to introduce two more matrix “basis elements”, so that four basis elements would completely represent the four elements (by addition) of a  $2 \times 2$  matrix. In a search for a name similar to “complex”, I

decided on “intricate”. This is not a new idea. I later learnt it was introduced by James Cockle in 1849, where he called these entities coquaternions. The modern terminology is often split-quaternions.

What I then realised was that the idea could be extended to  $2^n \times 2^n$  matrices, in a way that was a completely regular extension of the  $2 \times 2$  case, unlike the complicated classifications of Lie group theory, which is the usual approach (of course Lie groups as objects exist!) These I called “hyperintricate numbers”, by analogy with “hypercomplex numbers” introduced by Hamilton. I also realised that Euler’s relation  $e^{i\theta} = \cos \theta + i \sin \theta$ , had an analogue for intricate numbers, using cosh and sinh. What I was initially unable to do was come up with a formula for  $e^{p1 + qi + r\alpha + s\phi}$ , where the exponents are an intricate number. The realisation of the correct formula for this came much later – I had inconsistencies in dealing with intricate roots in Galois theory, and I am not sure on how I managed to jump to the correct formula.

On matrix Galois theory, where for complex polynomials there is always a matrix solution for a polynomial equation of arbitrary degree, in 2011 I made the following comments, which should not relate to complex coefficients, but intricate or hyperintricate ones. All this theory has now reached its adolescent stage, and there are quite a number of results. To continue with the Galois idea, I have now proved Galois restrictions hold for polynomials with J-abelian matrix variables, using an idea called J-abelian algebra. Further development of the paper on this, both for explicit solutions up to degree 4 equations, and the work on perturbation methods, will use this idea.

I could mention that Fermat’s little theorem (not the last theorem, although that is also of interest – there is work on Beal’s conjecture on the website) can be extended to hyperintricate numbers. I have extended results on division algebras so they deal with the sort of “non-standard” cases which fascinate me. I would also like to mention the work on hyperintricate exponentiation. This follows partly from a separate idea. I have always thought the currently accepted ideas are strange, and in my work on this I have *defined* them as inconsistent. I had the misfortune to invent an approach to  $i^i$  in answering an examination question at university. I proved this inconsistent, and it has perplexed and frustrated me ever since. I had a friend who sent it on to David Bohm, the physicist, who became interested because of its implications for the Riemann hypothesis. The work on hyperintricate exponentiation, which has now reached a stage of fruition, attempts to tackle this question in detail.

An idea I described which was recent in 2011 was a classification of nonassociative algebras by introducing new matrix operations using the hyperintricate representation. The reason I did this was because I could find no modification of standard matrix multiplication that produced the algebra of the octonions. Suspecting a fundamental obstruction to such a representation – as I had witnessed in determining solutions in noncommutative Galois theory, I invented the diamond and left and right roll operations, which describe in a very general way nonassociative intricate operations. I now suspect nonassociative applications to the octonion division algebra, Galois theory (I think nonassociative Galois theory might have some aspects of novelty), and hyperintricate exponential algebras.

There is now some work on novanion algebras in Superexponential algebra. These are division algebras except for the scalar part. When the scalar part is zero, they are not division algebras. An idea is to use them to describe physics. We say time is the scalar part, so outside time zero there are no operations which will annihilate states – energy is conserved.

There is some other work on the website. Thirty years ago I did some research on probability. This has now been written up, and more recent extensions have been added, finalised with work on the theory of sheaves and toposes. The category theory has now been removed, and this is part of the eBook Superexponential algebra.

As I said in 2011, also of interest (last but not least!) is the work on branched spaces. Once again, this is revolutionary (if you like revolutions, they are exactly the sort of thing you like!). These violate the “boundary of a boundary = 0” in some pathological cases (infinite explosions), which is embedded in the structure of the group approach to homology and cohomology. I have had to go back to the history of the topic (the work of Riemann, Betti, Klein, Poincaré, Seifert and Threlfall, Alexandroff and Hopf, Noether and Weyl are some of the early works) in order to disentangle what it is about. It is clear to me now that the work of Mac Lane, Eilenberg, Cartan and Grothendieck are not the beginning of the subject, but its mid-point. A remark I would like to make is that the historical material, before 1935, looks natural, and the later revolutions are pedagogically not the way one would wish to introduce it. Having completed the popularisation part of the work – part I, I am now in the process of introducing a revolution again – the whole basis of the theory in part II is a polynomial – which idea links with my other works, and it combines homology theory (called now chromology to distinguish it from the conventional ideas) and homotopy theory (called now chromotopy) together. What comes out is a complex polynomial for the Euler-Poincaré characteristic – with chromotopy supplying the imaginary part. However, currently the idea of boundary in general chromotopy does not seem to have the same significance, whereas in homology this is central, as are questions of topological equivalence of simplicial divisions, whereas general chromotopy is characterised by an expression related to one particular type of subdivision. This work with modifications is now under assembly in the eBook Superexponential algebra.

I will end with a list of further areas to which I believe hyperintricate numbers can have an influence in the development of subjects: Zeta functions, L series, Fourier and wavelet analysis, hyperintricate analysis, classification of groups, and noncommutative rings. Areas I have not mentioned are practical applications. It follows that since hyperintricate numbers contain complex numbers on one end and matrices on the other, and these mathematical tools abound in science and engineering, that this is a fertile ground for further developments and applications.