Mathematical Research Topics

Jim Adams 25th December 2017

Four books on mathematics are to be published, firstly as eBooks.
1. *Elementary methods in number theory*. This consists of early work dating mainly from 2009, collected together in one volume. It contains work on exponential powers, extensions of Fermat's little theorem, analyses elliptic curves (mod 4), introduces quadronacci numbers, which are an extended idea from Fibonacci and tribonacci numbers, discusses Fermat's last theorem from the point of view of elementary methods, and Beal's conjecture using similar methods.

2. *Innovation in mathematics* was mainly completed apart from marketing in August 2014, with the final update in February 2017. It has been on the Assayer free eBook website since November 2014. Some chapters have been developed much further in the eBook *Superexponential algebra*.

3. *Superexponential algebra*, in three volumes. Work was completed for volume I in January 2016, volume II in December 2016 and volume III in January 2017. The eBook was released for comments in September 2015, with a posting to the Assayer in October 2015. This eBook is a major teaching work of innovative research. Using Lagrange's four squares thorem and a classification proof for quaternions, we give a new proof of Wedderburn's little theorem, which states that finite division rings are commutative, introduce new ideas on novanions, show the ineffectiveness of Galois theory for dependent roots, matrix roots, with investigations of ring automorphisms deconstructing the Galois theory of group automorphisms, and give a replacement of the proof of the unsolvability of the quintic by killing central terms without using group theory. New results deconstructing the uncountable continuum hypothesis and on decidability are given, where we show that consistent problems are decidable. The eBook ascends from discussion mainly of rings, then to exponential structures and finally superexponential algebra. Logic is an integral part of this work. It also deals with probability logics, and amalgamates the previously intended eBooks *Algorithms and consistency* and *Algebra, space and logic*. The original intention was to include work on *Branched spaces*, but this has been hived off to the next item.

4. *Number, space and logic*. The number theory part of this work proves the Riemann hypothesis from the point of view Dw exponential algebras with w imaginary, first suggested by David Bohm in the 1980's. An intention is to prove the general Riemann hypothesis as an extension of the same techniques. The work includes a full description of the properties of the zeta function and looks at local and function fields in this context. We discuss M algebra, which is the transcendental analogue of the natural numbers, N. A realisable aspiration is to prove the Riemann hypothesis using M algebra. Extending the work on novanions, we look at the classification of simple groups from this point of view. We find the solution of the sextic by radicals by nondescending comparison methods. We will describe Betti numbers, and Leray spectral sequences will be needed, the latter being put in the more general context of branched spaces. We will describe l-adic cohomolgy (étale cohomology) and l-adic Fourier analysis to do this. An exposition of the Lefschetz fixed point theorem will be given.

We will complete the discussion of colour logics begun in *Superexponential algebra*, and the extended Gentzen proof of the consistency of analysis. An overarching idea in this work is the combination of explosion/implosion algebra with that of branched spaces, both for superexponential structures and topological generalisations, with logics related to these two, and for multivalued logics. The work has begun in May 2017.

23rd August 2014, revised 30th January 2016

Update. The eBook *Innovation in mathematics* is now complete apart from marketing. It contains a description of the creative process in mathematics, a 50 page piece on the inconsistency of the real number system and a replacement, ladder numbers, a work showing that the sextic polynomial with duplicate roots is solvable, an introduction to the intricate and hyperintricate representation of matrices, with a follow-up on the extricate and duplicate non-matrix algebras, a work giving a simple proof of the quadratic residues formula, extending cubic and quartic reciprocity to an extension of Eulers totient formula for arbitrary characteristic, two investigations on asymmetries of clock (congruence) arithmetic for quadratic residues mod a prime p = 4k – 1 and a translation of a work by Eisenstein.

The eBook *Superexponential algebra* is in progress. It contains a Prologue on why mathematics is there and second part describing what mathematics consists of. The chapters on Intricate numbers and Hyperintricate numbers are taken from *Innovation in mathematics*, the chapter on associative division algebras includes a proof that the maximum dimension of an associative division algebra is 4, a proof via Lagrange's four squares theorem of Wedderburn's little theorem that every finite division ring is abelian, and combining the two together, a short (three pages not a over a hundred) proof of the famous Feit-Thompson theorem. We distort matrix operations to provide new ones, and introduce division algebras for which we must also specify that the scalar part is nonzero – the n-novanions, and prove by eigenvalue methods that 10-novanions are consistent The chapter on Fermat's little theorem for matrices now includes a section on the Chinese eigenvalue remainder theorem. This goes up to chapter 6, and there will be 23 chapters in total.

11th November 2011, revised 10th November 2013

Introduction.

The following are research topics (with some indication of references) concerning intricate numbers and other subjects, in a research programme that I am progressing now on my own and for which I invite contributions and critiques.

Contributions are welcome at all levels of mathematical ability. If the number of researchers becomes large, there should be some way of allocating tasks so they are not duplicated. Your suggestions are welcome here, but that is for the future.

My own research spans four subject areas.

There are two papers on quadratic residues which use elementary methods in standard number theory. The main innovation consists of extending the Euclidean division algorithm so it is not unique. This has been useful in listing out quadratic residue characteristics for each row, where the rows are each a traversal of one cycle in mod arithmetic.
The hyperintricate representation is detailed in [Adams 2]. It is a representation of real $2^n \times 2^n$ matrices, the 2×2 matrix version of which (intricate numbers) contains as a subalgebra the complex numbers. The confluence of these two ideas – complex numbers and matrices – allows the development of mathematics from both these subject areas and their interaction. Its use in Galois theory for matrix variables is documented in [Adams 3] and [Adams 4], an application to Fermat's little theorem for matrices in [Adams 7], and to division algebras in [Adams 10]. Hyperintricate exponentiation is documented in the two papers [Adams 9], the second of which introduces a novel hyperintricate algebra for exponentiation. Hyperanalysis, which extends the ideas of complex analysis, is given in [Adams 12]. The hyperintricate research programme intersects with what appears to be new ideas on probability sheaves introduced in [Adams 8]. The hyperintricate methodology has now been extended to derive nonassociative operations from associative matrix operations [Adams 14], and may have applications to work on division algebras, exponential algebras and noncommutative Galois theory. I have recently (re-?)discovered a non-matrix algebra derived from multiplication tables of elements, and somewhat similar to the hyperintricate algebra (and more general than Kac-Moody algebras). This is called the hyperextricate algebra. When applied, the speculation is the new algebra is applicable to the octonions, sedenions and higher order algebras, and these modified algebras are indeed division algebras, unlike the standard sedenions, which are not.

The concept of branched retract spaces is documented in a number of papers, paper [Adams 5] being the only one that is ready. This is probably an innovation. The other articles develop the technical machinery of branched space theory.

The fourth subject area is a paper on analysis [Adams 16], which deconstructs the Cantor diagonal argument showing this is not generic, and uses transfinite induction to prove the inconsistency of analysis, as currently axiomatised, the algebra for which appears to be an innovation. The axioms of well-ordering are abandoned and with it the usual domain of application of induction. We are therefore not dealing with real numbers, and the term ladder number is adopted. The last part creates an algebra for zero different than the normal. Then 0/0 becomes a set, the set of real numbers. This topic will be developed further in a separate paper itemising the work of Gentzen, and tidying up the original paper by including a discussion of algorithms. Note that what Gentzen called transfinite induction is in terms of the ladder number paper countable induction.

These ideas have now reached a stage of interrelationship which will be developed.

The congruence arithmetic work in [Adams 15] can be applied to other areas of the research programme.

The ladder numbers work has been intended to be applied to the work on branched algebra. In particular trees represent free groups, so that infinite and transinfinite groups can be approached in this way. This includes the fact, present in homology theory, that for groups the boundary of a boundary is not zero, so that exact sequences for Ext and Tor have to be extended on the right or left. The ladder extension of real numbers to explosions contains the example in which $\partial\partial$ is not necessarily zero. Branched algebra contains the standard homotopical theory as a proper subset.
Further, we can deal with ladder number varieties, extended to hyperintricate ladder number varieties. Thus the whole programme initiated by Grothendieck comes into focus in a more general setting, that of the confluence of hyperintricate varieties, ladder numbers, branched algebra and what is itemised next.

The work on superexponential algebra [Adams 17] can be incorporated within this programme. The question of whether work on non-associative systems, which is a natural part of this algebra, is sufficiently rich to encompass the description of the octonions, can be posed.

The above research will be incorporated in three self-published books, promoted by social media. A proper, but informal, mathematical review process will be part of the assembly. Once these works have reached a sufficiently developed stage, copyright will be removed. These books will be: ‘Innovations in mathematics’, ‘Hyperintricate numbers’ and ‘Branched spaces’.

‘Innovations in mathematics’ will contain a chapter on the creative process in mathematical research and as examples the works on ‘Discussion on ladder numbers’, ‘Introduction to intricate and hyperintricate numbers’ I and II, ‘Some simple proofs on general reciprocity’, possibly the work on ‘Beal’s conjecture’, and ‘An elementary proof of the p = 4k – 1 asymmetry theorem for quadratic residues’ I and II.

‘Hyperintricate numbers’ will contain ‘Foundations’ and ‘Applications’ parts, which will be integrated with the ‘Rescaling linear and polynomial probabilities’ – which will be expanded to deal with multi-valued logics – and ‘Superexponentiation’ papers.

‘Branched spaces’ will be developed from the ‘Branched spaces’ I and II works, and those works currently in incomplete condition on the topic of chromotopic algebra (which will revert back to the branched spaces title). It will be developed to the level of say the author Jacob Lurie on spectral sequences, and will contain a discussion of quantum chromodynamics. The interrelationship of multi-valued logics to branched spaces will be discussed.

The following topics can be developed.

1. **<u>Hyperintricate Fermat’s little theorem.</u>**

The matrix case of Fermat’s little theorem is in [Adams 7]. Totients are detailed in [Conway and Guy] – theorems are obtained connecting the hyperintricate little theorem with totients, and extensions of these results are required.

First start with integers – Fermat’s little theorem works here. What you are doing is asserting for p prime that

$1^p - 1 \equiv 0 \pmod p$

for the initial induction hypothesis, and that

$y^p - y \equiv 0 \pmod p$

implies by the binomial theorem, since the denominators are not divisible by p,

$(y + 1)^p - (y + 1) \equiv 0 \pmod p$

recursively. This clearly works for integers, and having been proved for integers, the inductive extension to p odd prime for Gaussian integers is easy; as a second stage deal with the imaginary part, although this involves a modification for p = 4k – 1.

For intricates, the situation is a similar extension, to numbers of the form

$a1 + bi + c\alpha + d\phi$,

also for hyperintricates, as is now proved (but not for totients). One has to be careful with the hyperintricate multinomial theorem, since this does not carry over directly from the commutative case.

2. Division algebras.

The paper [Adams 10] currently only deals with associative division algebras, that is, matrices, including with possibly singular matrices. This needs extending to the non-associative case, for example, octonions. I mention in passing that sedenions [Wikipedia] are describable by the Cayley-Dickson construction. The relation between this non-associative construction and hyperintricates could be mapped out.

3. Hyperintricate reciprocity.

Further work needs to be done connecting the paper [Adams 11] with quadratic reciprocity – a reference is [Cassels].

4. Hyperintricate Fermat's last theorem.

From my own clumsy efforts, I do not think Fermat's last theorem should be approached in a way that is not equivalent to Taylor-Wiles. However, there is the question of whether this holds also for non-nilpotent matrices, and the (intricate to begin with) and hyperintricate methodology can be transferred from sources like [Modular Forms and FLT], which is the abelian part. Does FLT hold for Gaussian integers and hence for J-abelian? (but $\omega^5 + \omega^{10} = 1$). This is a big research project.

5. Hyperintricate Galois theory.

The current write-up is in [Adams 3], [Adams 4] and I like the treatment given in [Rotman 1]. I have proved there are no J-abelian hyperintricate solutions to the quintic. The intricate solution of the quartic is *not* given in [Adams 3], and should be computed.

To complete this work two strands are possible: I have computed intricate and hyperintricate roots of unity. The group theory – a suitable reference is [Alperin and Bell] should be extended to explore its connections with the Galois mapping. Secondly, perturbation techniques (giving transcendental solutions) need to be explored. The technique of hyperintricate parts is a constraint, which is obviated in classical solutions. There may be no algorithm splitting solutions into parts in general.

6. **Hyperintricate zeta functions and L-series.**

Zeta functions are introduced in a full way in [Edwards]. Books on L-series are [Davenport] (suggested by Richard Guy), [Langlands 1], [Langlands 2] and [Langlands 3]. The reader could also look at [Dirichlet]. I think this is partly expressing in hyperintricate terms what has been done already. [Adams9] introduces a different exponential algebra, which means the Riemann hypothesis can be re-examined in this context.

7. **Hyperintricate cyclotomics.**

A reference is [Washington]. There are interesting relationships with Bernoulli numbers [Kummer] and the class number.

8. **Hyperintricate Fourier series.**

There are connections here with class field theory [Tate], but hyperintricate Fourier series need developing specifically as well.

9. **Hyperintricate Diophantine toposes and hyperintricate schemes.**

For the uninitiated, instead of topos, I could have used the word set, and a topos is a categorical description, but extended and I think simpler, of the idea of a set. For those that like the work of [Grothendieck] there is much that can be done here. See also [Modular forms and FLT] and [Weil]. For Riemann-Roch see [Hirschfeld].

For schemes, (and this is relevant to Hochschild cohomolgy) see [Demazur and Gabriel] we note we are taking a different approach. Modules have an abelian group, whereas we are dealing with non-commutative matrices. Conventionally, rings act on the K group, so ideals are often discussed, whereas I myself haven't extended the theory in that direction – polynomials in X are represented by powers of the matrix X.

10. **Hyperintricate elliptic curves.**

See [Silverman] and [Silverman and Tate].

11. **Hyperintricate cohomology.**

The current reference is [Adams 5] (which is a naive introduction). There may be connections with the theory of motives [Artin]. I liked the simple approach given in [Rotman 2], but [Mac Lane 3] has insight, suitable for branched space development. For physicists [Adams 5] mentions an application to quark confinement.

Since sums of hyperintricate basis elements can be singular matrices, we can also define a homology which deals with this circumstance – this is an intricate homology theory that deals with singularities. Such a situation is also evident in the study of elliptic curves.

12. Hyperintricate homotopy.

The branched space version of this is being written, called chromotopy, where indications are given in [Adams 5]. If anyone wants to blaze a path like that of Serre, the equivalent of the homotopy theory of spheres (but hyperintricately for branched spaces we are not dealing with spheres) needs developing.

13. Hyperintricate powers and exponents.

Non-abelian exponential addition seems to be expressible only via the format, e.g.

$$e^h = e^{a1 + bi + c\alpha + d\phi} \neq e^{a1}e^{bi}e^{c\alpha}e^{d\phi}.$$

It is important to know m^n where m and n are hyperintricate basis elements, in all cases. Work is being developed in [Adams 9], including an evaluation of alternative exponential algebras in which $i^i \neq e^{-\pi/2 + 2\pi k}$, contrary to the standard theory. This is extended to the intricate and hyperintricate cases, and is nearing completion.

14. Hyperintricate superexponentiation.

This should be the practical end of work on a theory extending categories [Mac Lane 1], [Mac Lane 2] to non-associative structures.

There is the problem with knowing what the next operation after exponentiation (tetration – I use the symbol ⥣, but the standard is $\uparrow^2$) gives for i ⥣ i in right nesting. This has been solved in [Adams 17]. The reference [Adams 6] is just a mess and not written up (the philosophy is getting it wrong is the first stage in getting it right!).

15. Connections with Lie groups and the classification of simple groups.

The hyperintricate representation is an alternative (and simpler) representation to that of Lie groups. The connections between this representation and that of Lie groups needs to be fully mapped out. [Herstein] gives the representation for quaternions, related to spinors [E. Cartan]. All such representations need to be obtained, including E_6, E_7 and E_8, and hyperintricate representations for the sporadic simple groups.

16. Hasse and Hecke.

The references are [Hasse] and [Hecke]. These works need extending to the hyperintricate case, particularly what might be called 'group' reciprocity.

17. Hyperintricate forms.

Quadratic forms and higher degree forms are currently described in terms of complex numbers. The hyperintricate case needs investigating.

18. Hyperintricate newforms.

A reference to the standard case is [Modular forms and FLT].

19. Hyperintricate polynomial probabilities.

A reference to work on this is [Adams 8]. This work has been expanded to incorporate further concepts from category theory, but mostly not its language.

20. Hyperintricate analysis.

The hyperintricate Cauchy-Riemann equations can be written down, Möbius functions and the Cauchy formula also need to be addressed – a nice mini research topic. An excellent reference which almost completes this work is [Needham].

21. Hyperintricate p-adic numbers.

A reference for a starting point is [Cassels]. To be considered ought to be the Kronecker-Weber theorem and Hensel's lemma. Note that valuations are not satisfied by hyperintricate numbers – all that survives is $|a||b| = |ab|$.

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