Discussion on Fermat's last theorem and elementary methods

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Fermat's last theorem (FLT) remained unsolved for 350 years. It would indeed be surprising if the methods available to Fermat could now be shown to be successful in proving this theorem. Nevertheless, it is the case that for the substantial period during which this problem remained unresolved, the principal techniques were derived from Kummer theory, which if its ancestry were to be traced, had its source in Euler's solution of the theorem for p = 3 using the cubic and complex numbers. During this period, there was no intimation except from the mathematically naive, that elementary methods could be successful where the more sophisticated Kummer techniques had failed.

This point of view is emphasised in Manin and Panchiskin's 'Modern number theory', where they point out that it may be proved that some results using complex numbers cannot be attained by elementary methods. However, on this last comment, I have seen no proof of this assertion, which is true when analytical results are available to sophisticated methods but not elementary ones. Indeed, by use of the hyperintricate techniques in my eBook 'Innovation in mathematics' complex number techniques may be reduced to questions on $2^n \times 2^n$ matrices with real coefficients, more specifically 2×2 matrices, and even a subset of these. It thus appears on general grounds that techniques on complex numbers may be reduced to questions about some aspect of linear transformations on real numbers.

Some contemporary mathematicians – Harold Edwards is an example – question the assertion that hypersophisticated methods are necessary for such an ostensibly simple problem (at least to state).

It appears that previous techniques were unsuccessful because they did not appropriate new ideas that are necessary for a resolution of this problem. We therefore invert the method of solution and ask the following question: Is it possible to prove by elementary methods that every semi-stable elliptic curve is modular – the central assertion of the Taylor-Wiles proof of Fermat's last theorem – sometimes reformulated with wider scope as the 'modularity theorem'?

In order to develop a program of research to answer this question, it is relevant to ask what distinct techniques have been developed to tackle FLT, and how can they be reformulated in terms of elementary methods. Only after this has been accomplished does it seem sensible, given the difficulty of the problem, to ask whether any of these techniques are superfluous or can be trimmed.

The relevant topics are

- Galois theory
- Modular forms
- p-adic numbers
- Grossencharacters
- Elliptic curves.

Galois theory is a theory of solvability, and since FLT can be solved, it seems secure to assume that the extensions of Galois techniques necessary for solution of the problem revert to solvable cases. Indeed, Galois theory in solvable cases reduces to the study of linear transformations as an expression of these solutions.

Modular forms revert, possibly, to a simple feature of multiplicative groups: the presence of division. We therefore expect any elementary proof of FLT to use in some general sense the idea of division, and that if this is not present, it will not work.

For p-adic numbers, an idea might be to reduce this to algebra (mod p^n) – and this is developed in the work on Totient reciprocity (but we may only need Euler's totient theorem in its elementary generalisation) given in our work already quoted.

For Grossencharacters, the intricate methodology reduces 2×2 matrices to intricate numbers, the Grossencharacter part of which relates to the study of the actual part, denoted by α, of a matrix representation.

For elliptic curves, and FLT uses an explicit result on the Cremona tables – on cusp forms of weight 2 – it has recently occurred to me that some features of this general case are contained in my 2009 work on Exponentiation, and it is possible that these elementary methods can be developed further.

The situation now arises that, with work, it may indeed be possible to capture all necessary features of the contemporary proof of FLT by elementary methods. However, since elementary methods are usually more difficult to implement than sophisticated ones, the question may be raised as to whether this is desirable.