Algorithms and consistency

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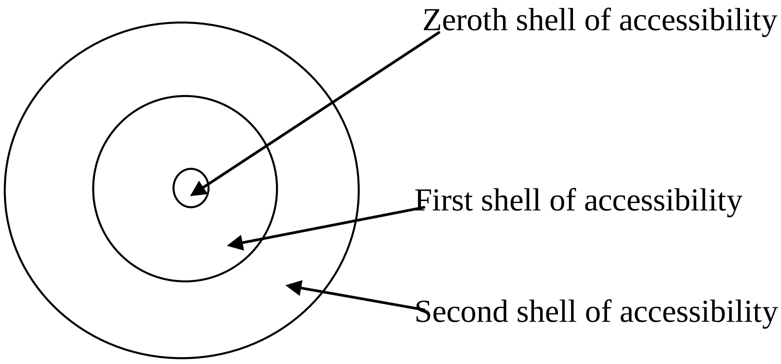
1. Introduction. This work is a sequel to 'Discussion on ladder numbers and the special zero algebra' [Ad14], and is due for completion in 2016.

2. Algorithms and accessibility. We will define an algorithm as a transformation or a collection of transformations over a set. This set may be finite if the algorithm halts after a finite number of steps, for example in a Turing machine. It will be countably infinite, as in the case for the type of induction, otherwise known as recursion, where the number of phases in a proof ranges over the natural numbers, ℕ. The range of induction may be a Eudoxus ladder number $\mathbf{L}_U$ of the form

$$... + a\boldsymbol{\varepsilon}^2 + b\boldsymbol{\varepsilon} + c + d\boldsymbol{\Omega} + e\boldsymbol{\Omega}^2 + ... \qquad (1)$$

where $\boldsymbol{\varepsilon}$ is an infinitesimal and $\boldsymbol{\Omega}$ an ordinal infinity in which $\boldsymbol{\varepsilon}.\boldsymbol{\Omega} = 1$ and $a, b, c, d, e \in \mathbb{U}$, where the set of Eudoxus numbers, $\mathbb{U}$, is a replacement for the reals. In [Ad14] the Cantor diagonal argument is shown to be ineffective, and $\mathbb{U}$ is described there as being countable. Further stages, of uncountability, are represented by the real ladder numbers $\mathbf{L}_R$, where the number of coefficients in (1) is uncountable.

An algorithm, depending on the set of transformations on which it is defined, may have various shells of accessibility.



Ordered by inclusion, a proof in shell n may not halt but the corresponding proof extended to shell (n + 1) may, if the cardinality M_{n+1} of its transformations is strictly greater than the smaller shell:

$$M_{n+1} > M_n.$$

Gentzen-type proof theory extends the shell of accessibility of $\mathbb{U}$, thereby proving the consistency of Eudoxus numbers, a situation unattainable by Gödel's incompleteness theorem in $\mathbb{U}$ itself. Our proof alsio extends Gentzen-type methods to superexponential techniques, inherent in the original Gentzen proof of the consistency of arithmetic.

3. Diagonal arguments. We have deconstructed the diagonal argument in [Ad14], and will need to investigate other areas of mathematics where a diagonal argument is employed. Examples are

(a) Gödel incompleteness.

(b) The word problem.

4. Globalisation in intuitionalistic logic. A shell is accessible at a higher value of n, or otherwise there exists a shell which differs from this (n + 1)th shell in the way it globalises. Thus

Standard globalisation ≠ nonstandard globalisation.

Intuitionalistic logics have an accessible Boolean globalisation, or this globalisation differs from the standard Boolean one (is m-valued). This m-valued logic may be accessible or inaccessible in some selection of the algorithmic extent and constraints. Then m-valued logics under globalisation differ from the Boolean as to

(a) Subobject classifier.

(b) Extended logical operations (&, OR, ⇒ (implies), ¬ (NOT)).

There may be other types of globalisation, but these are the only ones we consider.

5. Algorithms and irregularities. An algorithm implements a regularity (which might be defined by a function on a set, so the axiom of choice is present), which maps a set to its transfer. For example, a finite sequence might be specified by a formula. The strict transfer principle can apply this formula recursively to an infinite countable set like the natural numbers. This implements an algorithm (a finite one). When there is no algorithm from a finite set to this countable one, we call this an irregularity.

We have seen in the work *Superexponential algebra*, arising from the abandonment of Galois theory, but outside of its format the reinstatement of its end results from looking at varieties, that some finite polynomials are solvable and others are not. So we may have formulas corresponding to solvable systems or not. Does this mean we have solvable or unsolvable algorithms?

6. Inconsistency, statistics and probability logic. A mapping defined by a function from a domain to a codomain may mismatch another codomain. We can say the two codomains are mutually inconsistent, but if there is a distribution space of their values, we can measure how near the two codomains are. When the two codomains completely match, we have a value for the relative truth of one codomain to the other. When the two distributions are as far apart as possible, so that they are anticorrelated, we have another value measuring this anticorrelation. This correlation and anticorrelation may be given values in some number system.

We are assuming the function from the domain to the codomain is determinate and understood, where this specifies the rules of deduction from the domain to the codomain. This function itself may mismatch another function, and so we have a relative inconsistency between rules of deduction, which also can be measured.

Not only can we specify inconsistencies between states and between rules of deduction, we can also measure the spread of these values. Thus we have measures for logic which are not Boolean, nor are the rules for deduction considered in this sense determinate, but we may measure this indeterminacy precisely.

7. Superexponential logic. A logic we can consider by the above methods is implemented by states and rules of transformation existing in a superexponential algebra. The logic and rules of deduction can therefore be both noncommutative and nonassociative in ways that are not allowed in classical systems.

Novanion algebra and its superexponential extensions may thus describe such extended logics. It is a question of interest whether recursion itself may be derived from the existence of recursive games in the initial scalar zero (that is, time = 0) state, where creation of objects becomes possible from the zero state, but not elsewhere in the algebra.

8. Items to appear. A discussion of finite methods, p-adic and l-adic numbers, and as an example, a discussion of Fermat's Last theorem, will appear in the completed version of *Algorithms and consistency*.

References.

Ad14 J.H. Adams, *Discussion on ladder numbers and the special zero algebra*, in *Innovation in mathematics*, 2014.

Ad15 J.H. Adams, *Superexponential algebra*, 2015.

Ad16 J.H. Adams, *Pictures of physics, theories and experiments*, 2016.