CHAPTER VIII

The novanionic Borcherds construction

8.1 Introduction.

We will describe axiom systems which give mathematical structures for numbers. The intention is to provide the background for the discussion which follows.

On page 938 of 'Quantum field theory' by Eberhard Zeidler, with reference to the Thompson series, he writes
"Borcherds calculated this series using the monster Lie algebra. This Lie algebra is constructed as the space of physical states of a bosonic string moving in a $\mathbb{Z}_2$ orbifold $M/\mathbb{Z}_2$ of a 26-dimensional torus M".

By this means the classification of simple groups is derived. This classification is finite, according to conventional wisdom. We wish to investigate whether or not a spanner can be thrown in the works. The reader is invited to investigate the proof of the finiteness condition.

I now want to relate the above quote to the Hajas conjecture, which is fully developed in *New Physics* [Ad18] by Graham Ennis and me, in the mathematical chapter XVII developed for applications to universal physics.

If we are to accept the Hajas conjecture on the identification of the 10-dimensional heterotic string with 10-novanions, and the extension of this to a 26-dimensional bosonic string with the 26-novanions, then we note that the number of distinct n-novanionic algebras is infinite, given that the override condition in [Ad18] volume I chapter II can always be allocated, which corresponds non-trivially when the n-novanion contains octonionic components.

However, we note an interesting nonstandard conclusion we have arrived at with the Hajas identification, given essentially in [Ad18] chapter XI, namely that all n-novanions can be allocated either bosonic or fermionic parts. Thus both the 10- and 26-novanions may be allocated these components, and this also holds in general for n-novanions.

In chapter XX of [Ad18] we employ the 26-novanions in Heim theory extended to gluons and quarks. There exist other possible universes with $n > 26$. So the 26-novanions do indeed contain a bosonic algebra.

The conjecture is: can we apply the unbounded n-novanionic algebra to derive a Thompson series so that the number of simple groups is not bounded?

8.2 Novanion algebras.

We first give the axioms for a field, and successively weaken these axioms to give them for division algebras and novanion algebras.

The axioms for a *field* $\mathbb{F}$, +, ×, which we will denote simply by $\mathbb{F}$, satisfy for a, b, c ∈ $\mathbb{F}$, with a × b being written as ab

additive closure:	$a + b \in \mathbb{F}$	(1)
associativity:	$a + (b + c) = (a + b) + c$	(2)
abelian addition:	$a + b = b + a$	(3)

existence of a zero: there exists a $0 \in \mathbb{F}$ satisfying

$$a + 0 = a \tag{4}$$

existence of negative elements: there exists a $(-a) \in \mathbb{F}$ with

$$a + (-a) = 0, \tag{5}$$

which we write introducing subtraction as

$$a - a = 0$$

multiplicative closure: $ab \in \mathbb{F}$ (6)

associativity: $a(bc) = (ab)c$ (7)

commutativity: $ab = ba$ (8)

existence of a 1: there exists a $1 \in \mathbb{F}$ satisfying

$$a1 = a \tag{9}$$

existence of inverse elements: there exists an $a^{-1} \in \mathbb{F}$ for $a \neq 0$ with

$$a(a^{-1}) = 1, \tag{10}$$

which we write introducing division as

$$a/a = 1$$

distributive law: $a(b + c) = (ab) + (ac)$. (11)

An associative division algebra satisfies all axioms for a field except the multiplicative commutativity rule (8). A nonassociative division algebra, D, also drops the multiplicative associative rule (7). It may introduce the following axiom:

There do not exist $a, b \neq 0 \in D$ with

$$ab = 0. \tag{12}$$

This rule is unnecessary for associative division algebras, since if $1 \neq 0$ then (12) implies

$$ab \neq c,\ c \neq 0,$$

and therefore there exists no

$$a(b/c) = 1,$$

but b/c can be chosen to be the multiplicative inverse of a, a contradiction. ☐

A novanion algebra B drops axiom (12). It may substitute the following rule:

If $a^2, b^2 > 0$ for $a, b \in B$ and $a \in c$, $b \in d \in B$, there do not exist any c, d with

$$cd = 0. \tag{13}$$

We can provide existence proofs for these structures. The complex numbers constitute a field. The quaternions form an associative division algebra. The octonions form a nonassociative division algebra. As we have proved in [Ad15], vol. I, chapter V, the 10-novanions, for example, form a novanion algebra. ☐

The consistency of the complex numbers is contingent on the consistency of analysis, given as a Gentzen-type proof in chapter III of this volume. The consistency of the quaternions as an associative division algebra is given in the previous reference to [Ad15], as is also the consistency of the 10-novanions as a novanion algebra. The consistency of the octonions now follows from their nature as novanion algebras of a specific type. ☐

8.3 The classification of novanions.

8.4 The Borcherds construction.

8.5 Novanionic Borcherds constructions.

8.6. Conclusion.

Our mathematical journey has now attained lift-off and the whole Earth lies in perspective beneath us. But this is not the end of our journey, it is the beginning. The reader is invited to continue our exploration. Mathematically the whole universe lies before us.