CHAPTER I

Trees

1.1. Introduction.

1.2. Trees.

We first define trees [Se00].

Definition 1.2.1. A *graph* consists of a set X of *vertices* and a set Y of *edges*, with two mappings

$Y \to X \times X$: $y \to (\text{origin}(y), \text{terminus}(y))$

$Y \to Y$: $y \to \text{reverse}(y)$

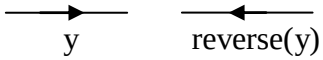
which for every $y \in Y$ satisfy the conditions

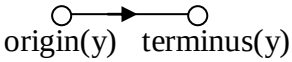
reverse $y \neq y$,

$\text{reverse}(\text{reverse}(y)) = y$

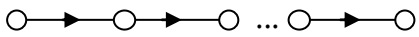
and

$\text{origin}(y) = \text{terminus}(\text{reverse}(y))$.





Definition 1.2.2. A *path* is a finite sequence of edges $y_1, y_2, ..., y_n$ with $\text{terminus}(y_i) = \text{origin}(y_{i+1})$, $i < n$. The path may be said to have an origin, terminus pair (y_1, y_n).



Definition 1.2.3. A *circuit* is a path with $\text{terminus}(y_n) = \text{origin}(y_1)$.

A circuit remains a circuit under a cyclic permutation of the y_i, since the circuit may be defined from a new path with origin y_k for some k.

Definition 1.2.4. A graph is said to be *connected* if any pair of vertices is the origin, terminus pair of at least one path.

Definition 1.2.5. A *tree* is a connected nonempty graph without circuits.

Definition 1.2.6. A *node* is an origin or a terminus in a tree. A *parent node* or *parent* is an origin in a tree. A *child node* or *child* is a terminus in a tree. A *root* of a tree is a child with no parent, a *leaf* of a tree is a parent with no child.