My mathematical journey

by Jim Adams, 11th August 2011

It is premature to give a history of the hyperintricate representation of matrices, but a colleague has suggested I give some historical background of my mathematical journey, so that the work can be put in context.

I don't have a degree, and my work on mathematics started in earnest in 2008 at the age of 58, after my career as a fairly senior payroll analyst programmer at P&O Properties had finished.

I introduced myself to groups and topology at the age of 15, and to general relativity at the age of 18. I have been a passive reader of advanced texts in mathematics and theoretical physics for a long time.

The situation became more active after I had learnt what it meant to compose music, to have my compositions performed, and even – hoisted by my own petard when I was organising a concert, and the pianist for my work withdrew – playing my own compositions myself to a live audience. This taught me that it was possible to produce music in a style that I wished to present – nudging at boundaries technically, in terms of the sonorities, and in the message that I occasionally wanted to project – yet at the same time I had little or no formal training. In this I was assisted by New Music Brighton, a self help group of composers, but it still took 5 years to get my first composition performed. To a certain extent, I was surprised by my own success. It showed me that it is possible to do things out of the ordinary, and yet be an ordinary person. That is how I developed the courage to embark on mathematical research.

If you look at my website – well some of the early stuff is suitably brushed under the carpet – you will see there are what is advertised as over 100 theorems on exponentiation. What I did, was start on something that I knew how to manipulate, and hoped I could produce results in – number theory. This is not very fashionable nowadays – the heyday of number theory was the 19th century (or even before). There are no major surprises in this work, except the demonstrated capacity to deal with detail, and to carry an idea to its technical exhaustion.

The first major result was what I thought would be a fairly simple task – solving an unsolved problem in what are called quadratic residues, using elementary methods – that is, not using complex numbers. A quadratic residue is the square of a number in clock arithmetic, in this case 12 o'clock is 0 and there are a prime number of hours. The problem I chose has been solved by other, transcendental methods, but it is stated by Richard Guy as an unsolved problem in his "Unsolved problems in number theory" book. I thought it would take me a month to solve it, but I had bitten off almost more than I could chew. It took me 18 months, other tasks intervened, and loads of calculation had to be performed. Eventually I "cheated". I looked at Hermann Weyl's proof of the transcendental result, and adopted part of it, Minkowski's lattice theory, so that its contents, but not form, were present in my proof.

I have been interested in my life in a number of mathematical ideas. One that caused me grief at university was the consistency of the real number system. This doesn't cause me problems nowadays, because I am acquainted with intuitionalistic logic, and I am aware there are several components to the axiom system for real numbers, like for example the continuum hypothesis and the "Archimedean" axiom that any real number can be multiplied by a natural number to be greater than any natural number – this was known to Euclid. The real numbers do not have to conform to this, as was known to Poincaré.

Another "problem" for me has been understanding Galois theory. The concrete part of this is that no polynomial of degree greater than 4 is "solvable by radicals", and what I didn't see, and originally I didn't understand the proof, was where the information was being "lost" so that the problem was unsolvable. Because I didn't understand the proof, I was trying the impossible – to solve the quintic. However, I believe the underlying question is still valid. If the underlying reason is group theory – which can be reduced to permutations, and if I can generalise numbers so that a permutation can be represented by one of these numbers – this isn't the case for complex numbers or quaternions – then can I solve the quintic. A recent realisation is that what I call substitution methods are ineffective in producing solutions, and this is because the group theory is multiplicative, and the multiplicative part is to do with powers of numbers, so I have to factorise 5 into something smaller (as was stated by Galois), and this *is* possible, $5 = (2 + i)(2 - i)$, but I don't think that solves the quintic, we have to go into non-commutative numbers, which I will mention next, so that we can map onto permutations of roots.

Well, apart from work on division algebras, and the work of my namesake, J. F. Adams, on this in the 1960's, what motivated me to introduce the hyperintricate representation was the search for a number system that was non-commutative, as I thought was necessary to solve "the impossible", by Galois theory. I wanted something that would incorporate complex numbers at one end, but also be non-commutative. The idea came from R. Remmert in a book called Numbers (by Ebbinghaus et al), although I had looked at the matrix representation of quaternions in a book by Herstein before this. It gave the matrix representation of complex numbers. I immediately realised that an extension of this idea was possible – to introduce two more matrix "basis elements", so that four basis elements would completely represent the four elements (by addition) of a 2 x 2 matrix. In a search for a name similar to "complex", I decided on "intricate".

What I then realised was that the idea could be extended to $2^n \times 2^n$ matrices, in a way that was a completely regular extension of the 2 x 2 case, unlike the complicated classifications of Lie group theory, which is the usual approach (of course Lie groups as objects exist!) These I called "hyperintricate numbers", by analogy with "hypercomplex numbers" introduced by Hamilton. I also realised that Euler's relation $e^{i\theta} = \cos\theta + i \sin\theta$, had an analogue for intricate numbers, using cosh and sinh. What I was initially unable to do was come up with a formula for $e^{p1 + qi + r\alpha + s\phi}$, where the exponents are an intricate number. The realisation of the correct formula for this came much later – I had inconsistencies in dealing with intricate roots in Galois theory, and I am not sure on how I managed to jump to the correct formula.

All this theory has now reached its adolescent stage, and there are quite a number of results. I could mention that Fermat's little theorem (not the last theorem, although that is also of interest) can be extended to hyperintricate numbers. I have extended results on division algebras so they deal with the sort of "non-standard" cases which fascinate me. I would also like to mention the work on hyperintricate exponentiation. This follows partly from a separate idea. I have always thought the currently accepted ideas are strange, and in my work on this I have *defined* them as inconsistent. I had the misfortune to invent an approach to i^i in answering an examination question at university. I proved this inconsistent, and it has perplexed and frustrated me ever since. I had a friend who sent it on to David Bohm, the physicist, who became interested. The paper on hyperintricate exponentiation, which may be reaching a stage of fruition, attempts to tackle this question in detail.

There is some other work on the website. Thirty years ago I did some research on probability. This has now been written up, and more recent extensions have been added.

Also of interest (last but not least!) is the work on branched spaces. Once again, this is revolutionary (if you like revolutions, they are exactly the sort of thing you like!). These violate the "boundary of a boundary = 0" in some cases, which is embedded in the structure of the group approach to homology and cohomology. I have had to go back to the history of the subject (the work of Riemann, Betti, Klein, Poincaré, Seifert and Threlfall, Alexandroff and Hopf, Noether and Weyl are some of the early works) in order to disentangle what the subject is about. It is clear to me now that the work of Mac Lane, Eilenberg, Cartan and Grothendieck are not the beginning of the subject, but its mid-point. A remark I would like to make is that the historical material, before 1935, looks natural, and the later revolutions are pedagogically not the way one would wish to introduce the subject.

I will end with a list of further areas to which I believe hyperintricate numbers can have an influence in developing subjects: Zeta functions, L series, Fourier analysis, Hyperintricate analysis, Hyperintricate homotopy, classification of groups. Areas I have not mentioned are practical applications. It follows that since hyperintricate numbers contain complex numbers on one end and matrices on the other, and these mathematical tools abound in science and engineering, that this is a fertile ground for further developments and applications.