Intricate Research Topics

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Introduction.

The following are research topics (with some indication of references) concerning intricate numbers.

Contributions are welcome at all levels of mathematical ability. If the number of researchers becomes large, there should be some way of allocating tasks so they are not duplicated. Your suggestions are welcome here, but that is for the future.

The hyperintricate representation is detailed in [Adams 2], and its use in Galois theory in [Adams 3] and [Adams 4].

The following topics can be developed.

1. Hyperintricate Fermat's little theorem.

The matrix case of Fermat's little theorem is in [Adams 7]. Totients are detailed in [Conway and Guy] – theorems are obtained connecting the hyperintricate little theorem with totients, and extensions of these results are required.

First start with integers – Fermat's little theorem works here. What you are doing is asserting for p prime that

$$1^p - 1 \equiv 0 \pmod p$$

for the initial induction hypothesis, and that

$$y^p - y \equiv 0 \pmod p$$

implies by the binomial theorem, since the denominators are not divisible by p,

$$(y + 1)^p - (y + 1) \equiv 0 \pmod p$$

recursively. This clearly works for integers, and having been proved for integers, the inductive extension to p odd prime for Gaussian integers is easy; as a second stage deal with the imaginary part, although this involves a modification for p = 4k – 1.

For intricates, the situation is a similar extension, to numbers of the form

$$a1 + bi + c\alpha + d\phi,$$

also for hyperintricates, as is now proved (but not for totients). One has to be careful with the hyperintricate multinomial theorem, since this does not carry over directly from the commutative case.

2. Division algebras.

The paper [Adams 10] currently only deals with associative division algebras, that is, matrices, including with possibly singular matrices. This needs extending to the non-associative case, for example, octonions. I mention in passing that sedenions [Wikipedia] are describable by the Cayley-Dickson construction. The relation between this non-associative construction and hyperintricates could be mapped out.

3. **Hyperintricate reciprocity.**

Further work needs to be done connecting the paper [Adams 11] with quadratic reciprocity – a reference is [Cassels].

4. **Hyperintricate Fermat's last theorem.**

From my own clumsy efforts, I do not think Fermat's last theorem should be approached in a way that is not equivalent to Taylor-Wiles. However, there is the question of whether this holds also for non-nilpotent matrices, and the (intricate to begin with) and hyperintricate methodology might be useful. This is a big research project.

5. **Hyperintricate Galois theory.**

The current write-up is in [Adams 3], [Adams 4] and I like the treatment given in [Rotman 1]. I have now proved there are no hyperintricate solutions to the quintic. Hyperintricate solutions are equivalent to intricate, and therefore to real ones. The intricate solution of the quartic is *not* given in [Adams 3], and should be computed.

To complete this work two strands are possible: I have computed intricate and hyperintricate roots of unity. The group theory – a suitable reference is [Alperin and Bell] should be extended to explore its connections with the *compression* technique, which provides the Galois mapping. Secondly, perturbation techniques (giving transcendental solutions) need to be explored. A problem is that the technique of hyperintricate parts is an additional constraint, which is obviated in classical solutions. Thus there may be no algorithm splitting solutions into parts in general.

6. **Hyperintricate zeta functions and L-series.**

Zeta functions are introduced in a full way in [Edwards]. Books on L-series are [Davenport] (suggested by Richard Guy), [Langlands 1], [Langlands 2] and [Langlands 3]. The reader could also look at [Dirichlet]. I think this is practically virgin territory from the hyperintricate aspect.

7. **Hyperintricate cyclotomics.**

A reference is [Washington]. There are interesting relationships with Bernoulli numbers [Kummer] and the class number.

8. **Hyperintricate Fourier series.**

There are connections here with class field theory [Tate], but hyperintricate Fourier series need developing specifically as well.

9. **Hyperintricate Diophantine toposes and hyperintricate schemes.**

For the uninitiated, instead of topos, I could have used the word set, and a topos is a categorical description, but extended and I think simpler, of the idea of a set. For those that like the work of [Grothendieck] there is much that can be done here. See also [Modular forms and FLT] and [Weil]. For Riemann-Roch see [Hirschfeld].

For schemes, (and this is relevant to Hochschild cohomolgy) see [Demazur and Gabriel] we note we are taking a different approach. Modules have an abelian group, whereas we are dealing with non-commutative matrices. Conventionally, rings act on the K group, so ideals are often discussed, whereas I myself haven't extended the theory in that direction – polynomials in X are represented by powers of the matrix X.

10. Hyperintricate elliptic curves.

See [Silverman] and [Silverman and Tate].

11. Hyperintricate cohomology.

The current reference is [Adams 5] (which is a naive introduction). There may be connections with the theory of motives [Artin]. I liked the simple approach given in [Rotman 2], but [Mac Lane 3] has insight, suitable for branched space development. For physicists [Adams 5] mentions an application to quark confinement.

Since sums of hyperintricate basis elements can be singular matrices, we can also define a homology which deals with this circumstance – this is an intricate homology theory that deals with singularities. Such a situation is also evident in the study of elliptic curves.

12. Hyperintricate homotopy.

Nothing has yet been done on this, except I suppose indications are given in [Adams 5]. If anyone wants to blaze a path like that of Serre, the equivalent of the homotopy theory of spheres (but hyperintricately we are not dealing with spheres) needs developing.

13. Hyperintricate powers and exponents.

Non-abelian exponential addition seems to be expressible only via the format, e.g.

$$e^h = e^{a1 + bi + c\alpha + d\phi} \neq e^{a1}e^{bi}e^{c\alpha}e^{d\phi}.$$

It is important to know m^n where m and n are hyperintricate basis elements, in all cases. Work is being developed in [Adams 9], including an evaluation of alternative exponential algebras in which $i^i \neq e^{-\pi/2 + 2\pi k}$, contrary to the standard theory. This is extended to the intricate and hyperintricate cases, and is nearing completion.

14. Hyperintricate superexponentiation.

This should be the practical end of work on a theory extending categories [Mac Lane 1], [Mac Lane 2] to non-associative structures.

There are problems with knowing what the next operation after exponentiation (tetration – I use the symbol ⇈, but the standard is $\uparrow^2$) gives for i ⇈ i in right nesting. The reference [Adams 6] is just a mess and not written up (the philosophy is getting it wrong is the first stage in getting it right!).

15. Connections with Lie groups and the classification of simple groups.

The hyperintricate representation is an alternative (and simpler) representation to that of Lie groups. The connections between this representation and that of Lie groups needs to be fully mapped out. [Herstein] gives the representation for quaternions, related to spinors [E. Cartan]. All such representations need to be obtained, including E_6, E_7 and E_8, and hyperintricate representations for the sporadic simple groups.

16. Hasse and Hecke.

The references are [Hasse] and [Hecke]. These works need extending to the hyperintricate case, particularly what might be called ‘group’ reciprocity.

17. Hyperintricate forms.

Quadratic forms and higher degree forms are currently described in terms of complex numbers. The hyperintricate case needs investigating.

18. Hyperintricate newforms.

A reference to the standard case is [Modular forms and FLT].

19. Hyperintricate polynomial probabilities.

A reference to work on this is [Adams 8]. This work has been expanded to incorporate further concepts from category theory, but mostly not its language.

20. Hyperintricate analysis.

The hyperintricate Cauchy-Riemann equations can be written down, Möbius functions and the Cauchy formula also need to be addressed – a nice mini research topic. An excellent reference which almost completes this work is [Needham].

21. Hyperintricate p-adic numbers.

A reference for a starting point is [Cassels]. To be considered ought to be the Kronecker-Weber theorem and Hensel’s lemma. Note that valuations are not satisfied by hyperintricate numbers – all that survives is $|a||b| = |ab|$.

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