Chapter XXI
Nonstandard accounts of physics

21.1 Introduction.

21.2 The experiments of Brush.

21.3 Wheatstone.

21.4 Rayleigh.

21.5 MOND.

21.6 Gyroscopes.

We will consider two coordinate systems, the second of which is rotating with respect to the first. The first or primary coordinate system is an inertial frame, that is, it is moving with uniform velocity with respect to a stationary reference frame, however defined.

Consider a vector **r** in the first frame, and an increment $\delta\mathbf{r}$ in its value. In the rotating frame if the corresponding vectors are $\mathbf{r}_{rot}$ and $\delta\mathbf{r}_{rot}$,

General derivation[edit]

Many problems require use of noninertial reference frames, for example, those involving satellites[21][22] and particle accelerators.[23] Figure 2 shows a particle with mass *m* and position vector $\mathbf{x}_A(t)$ in a particular inertial frame A. Consider a non-inertial frame B whose origin relative to the inertial one is given by $\mathbf{X}_{AB}(t)$. Let the position of the particle in frame B be $\mathbf{x}_B(t)$. What is the force on the particle as expressed in the coordinate system of frame B? [24][25]

To answer this question, let the coordinate axis in B be represented by unit vectors $\mathbf{u}_j$ with *j* any of { 1, 2, 3 } for the three coordinate axes. Then

The interpretation of this equation is that $\mathbf{x}_B$ is the vector displacement of the particle as expressed in terms of the coordinates in frame B at time *t*. From frame A the particle is located at:

As an aside, the unit vectors { $\mathbf{u}_j$ } cannot change magnitude, so derivatives of these vectors express only rotation of the coordinate system B. On the other hand, vector $\mathbf{X}_{AB}$ simply locates the origin of frame B relative to frame A, and so cannot include rotation of frame B.

Taking a time derivative, the velocity of the particle is:

The second term summation is the velocity of the particle, say $\mathbf{v}_B$ as measured in frame B. That is:

The interpretation of this equation is that the velocity of the particle seen by observers in frame A consists of what observers in frame B call the velocity, namely $\mathbf{v}_B$, plus two extra terms related to the rate of change of the frame-B coordinate axes. One of these is simply the velocity of the moving origin $\mathbf{v}_{AB}$. The other is a contribution to velocity due to the fact that different locations in the non-inertial frame have different apparent velocities due to rotation of the frame; a point seen from a rotating frame has a rotational component of velocity that is greater the further the point is from the origin.

To find the acceleration, another time differentiation provides:

Using the same formula already used for the time derivative of $\mathbf{x}_B$, the velocity derivative on the right is:

Consequently,

(*1*)

The interpretation of this equation is as follows: the acceleration of the particle in frame A consists of what observers in frame B call the particle acceleration $\mathbf{a}_B$, but in addition there are three acceleration terms related to the movement of the frame-B coordinate axes: one term related to the acceleration of the origin of frame B, namely $\mathbf{a}_{AB}$, and two terms related to rotation of frame B. Consequently, observers in B will see the particle motion as possessing "extra" acceleration, which they will attribute to "forces" acting on the particle, but which observers in A say are "fictitious" forces arising simply because observers in B do not recognize the non-inertial nature of frame B.

The factor of two in the Coriolis force arises from two equal contributions: (i) the apparent change of an inertially constant velocity with time because rotation makes the direction of the velocity seem to change (a $d\mathbf{v}_B/dt$ term) and (ii) an apparent change in the velocity of an object when its position changes, putting it nearer to or further from the axis of rotation (the change in due to change in x_j).

To put matters in terms of forces, the accelerations are multiplied by the particle mass:

The force observed in frame B, $\mathbf{F}_B = m\mathbf{a}_B$ is related to the actual force on the particle, $\mathbf{F}_A$, by

where:

Thus, we can solve problems in frame B by assuming that Newton's second law holds (with respect to quantities in that frame) and treating $\mathbf{F}_{fictitious}$ as an additional force.[12][26][27]

Below are a number of examples applying this result for fictitious forces. More examples can be found in the article on centrifugal force.

Rotating coordinate systems[edit]

A common situation in which noninertial reference frames are useful is when the reference frame is rotating. Because such rotational motion is non-inertial, due to the acceleration present in any rotational motion, a fictitious force can always be invoked by using a rotational frame of reference. Despite this complication, the use of fictitious forces often simplifies the calculations involved.

To derive expressions for the fictitious forces, derivatives are needed for the apparent time rate of change of vectors that take into account time-variation of the coordinate axes. If the rotation of frame 'B' is represented by a vector $\mathbf{\Omega}$ pointed along the axis of rotation with orientation given by the right-hand rule, and with magnitude given by

then the time derivative of any of the three unit vectors describing frame B is[26][28]

and

as is verified using the properties of the vector cross product. These derivative formulas now are applied to the relationship between acceleration in an inertial frame, and that in a coordinate frame rotating with time-varying angular velocity $\omega(t)$. From the previous section, where subscript A refers to the inertial frame and B to the rotating frame, setting $\mathbf{a}_{AB} = 0$ to remove any translational acceleration, and focusing on only rotational properties (see Eq. 1):

Collecting terms, the result is the so-called *acceleration transformation formula*:[29]

The [physical acceleration](#) $\mathbf{a}_A$ due to what observers in the inertial frame A call *real external forces* on the object is, therefore, not simply the acceleration $\mathbf{a}_B$ seen by observers in the rotational frame B, but has several additional geometric acceleration terms associated with the rotation of B. As seen in the rotational frame, the acceleration $\mathbf{a}_B$ of the particle is given by rearrangement of the above equation as:

The net force upon the object according to observers in the rotating frame is $\mathbf{F}_B = m\mathbf{a}_B$. If their observations are to result in the correct force on the object when using Newton's laws, they must consider that the additional force $\mathbf{F}_{fict}$ is present, so the end result is $\mathbf{F}_B = \mathbf{F}_A + \mathbf{F}_{fict}$. Thus, the fictitious force used by observers in B to get the correct behavior of the object from Newton's laws equals:

Here, the first term is the *Coriolis force*,[30] the second term is the *centrifugal force*,[31] and the third term is the *Euler force*.[32][33]

Orbiting coordinate systems[edit]

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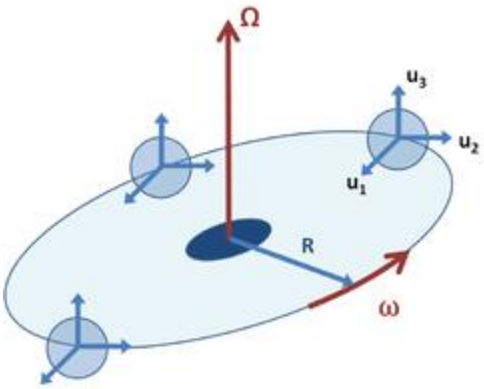

Figure 3: An orbiting but fixed orientation coordinate system *B*, shown at three different times. The unit vectors $\mathbf{u}_j$, j = 1, 2, 3 do *not* rotate, but maintain a fixed orientation, while the

origin of the coordinate system *B* moves at constant angular rate ω about the fixed axis **Ω**. Axis **Ω** passes through the origin of inertial frame *A*, so the origin of frame *B* is a fixed distance *R* from the origin of inertial frame *A*.

As a related example, suppose the moving coordinate system *B* rotates in a circle of radius *R* about the fixed origin of inertial frame *A*, but maintains its coordinate axes fixed in orientation, as in Figure 3. The acceleration of an observed body is now (see Eq. 1):

where the summations are zero inasmuch as the unit vectors have no time dependence. The origin of system *B* is located according to frame *A* at:

leading to a velocity of the origin of frame *B* as:

leading to an acceleration of the origin of *B* given by:

Because the first term, which is

is of the same form as the normal centrifugal force expression:

it is a natural extension of standard terminology (although there is no standard terminology for this case) to call this term a "centrifugal force". Whatever terminology is adopted, the observers in frame *B* must introduce a fictitious force, this time due to the acceleration from the orbital motion of their entire coordinate frame, that is radially outward away from the center of rotation of the origin of their coordinate system:

and of magnitude:

Notice that this "centrifugal force" has differences from the case of a rotating frame. In the rotating frame the centrifugal force is related to the distance of the object from the origin of frame *B*, while in the case of an orbiting frame, the centrifugal force is independent of the distance of the object from the origin of frame *B*, but instead depends upon the distance of the origin of frame *B* from *its* center of rotation, resulting in the *same* centrifugal fictitious force for *all* objects observed in frame *B*.

Orbiting and rotating[edit]

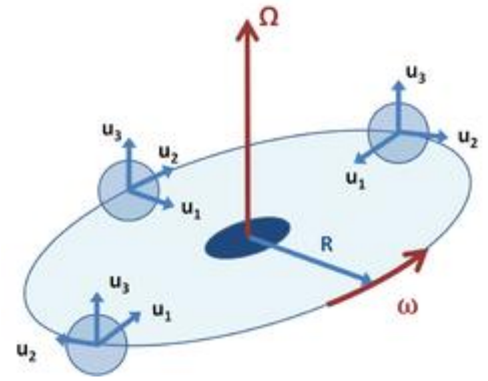

Figure 4: An orbiting coordinate system *B* similar to Figure 3, but in which unit vectors $\mathbf{u}_j$, j = 1, 2, 3 rotate to face the rotational axis, while the origin of the coordinate system *B* moves at constant angular rate ω about the fixed axis **Ω**.

As a combination example, Figure 4 shows a coordinate system *B* that orbits inertial frame *A* as in Figure 3, but the coordinate axes in frame *B* turn so unit vector $\mathbf{u}_1$ always points toward the center of rotation. This example might apply to a test tube in a centrifuge, where vector $\mathbf{u}_1$ points along the axis of the tube toward its opening at its top. It also resembles the Earth-Moon system, where the Moon always presents the same face to the Earth.[34] In this example, unit vector $\mathbf{u}_3$ retains a fixed orientation, while vectors $\mathbf{u}_1$, $\mathbf{u}_2$ rotate at the same rate as the origin of coordinates. That is,

Hence, the acceleration of a moving object is expressed as (see Eq. 1):

where the angular acceleration term is zero for constant rate of rotation. Because the first term, which is is of the same form as the normal centrifugal force expression:

it is a natural extension of standard terminology (although there is no standard terminology for this case) to call this term the "centrifugal force". Applying this terminology to the example of a tube in a centrifuge, if the tube is far enough from the center of rotation, $|\mathbf{X}_{AB}| = R \gg |\mathbf{x}_B|$, all the matter in the test tube sees the same acceleration (the same centrifugal force). Thus, in this case, the fictitious force is primarily a uniform centrifugal force along the axis of the tube, away from the center of rotation, with a value $|\mathbf{F}_{Fict}| = \omega^2 R$, where R is the distance of the matter in the tube from the center of the centrifuge. It is standard specification of a centrifuge to use the "effective" radius of the centrifuge to estimate its ability to provide centrifugal force. Thus, a first estimate of centrifugal force in a centrifuge can be based upon the distance of the tubes from the center of rotation, and corrections applied if needed.[35][36]

Also, the test tube confines motion to the direction down the length of the tube, so $\mathbf{v}_B$ is opposite to $\mathbf{u}_1$ and the Coriolis force is opposite to $\mathbf{u}_2$, that is, against the wall of the tube. If the tube is spun for a long enough time, the velocity $\mathbf{v}_B$ drops to zero as the matter comes to an equilibrium distribution. For more details, see the articles on sedimentation and the Lamm equation.

A related problem is that of centrifugal forces for the Earth-Moon-Sun system, where three rotations appear: the daily rotation of the Earth about its axis, the lunar-month rotation of the Earth-Moon system about their center of mass, and the annual revolution of the Earth-Moon system about the Sun. These three motions influence the tides.[37]

21.7 Hooper.

21.8 Ball lightening.

21.9 Quantum gravity.