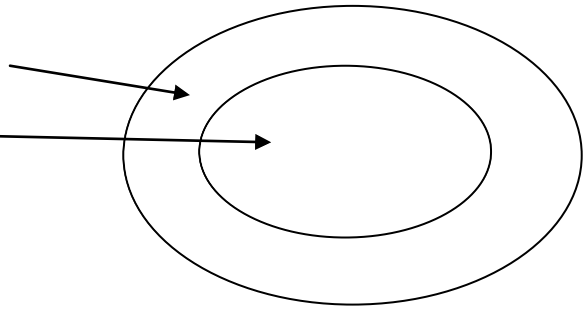
Prologue

To show how we link very general ideas to the specific way they are presented later in this work, the prologue is shown as the outer to the inner layers of the diagram

1. Why is mathematics there?

2. What is mathematics?



Why is mathematics there? [3We66]

From an early age, we are taught to identify number. The objects we describe with numbers, like apples, are only semi-permanent, so it is good to ask whether there exists in the objective world quantities or aspects which correspond to numbers that are absolutely conserved. Arising from our partial knowledge of the laws of physics in the present era, we can identify where numbers occur in nature, and their meaning, that is, their mapping to objective reality. But since our knowledge is incomplete, it is still relevant to ask whether as an absolute truth numbers really exist in the world.

For allocations in physics of the baryon number, lepton number or charm of a particle, if we ask whether these are conserved quantities or not, it may be posed as a philosophical problem that if no physical variable mapped onto a number is conserved, does its non-existence in the universe imply in an absolute sense that we cannot correctly use numbers at all?

So there is a divide between the physical world and the world of abstract reasoning, and we have raised the question as to whether this gap can be bridged. It is my point of view that we can use reasoning whose basis is secure without the need that the objects of the reasoning or the allowable rules of the system of reasoning is available in the physical world.

If this is so, and it is a possible starting point, the question arises as to whether mathematics resides only in its symbols and systems of writing, and is not necessarily inherent in nature.

But we may ask why at least approximations to the idea of number exist in the real world. This, like all theories of physics, can only be answered by a speculation. These speculations are thought appropriate when they have enormous explanatory power and massive confirmation in physical experiments, but there is usually a phase in leading up to the acceptance of a theory where explanatory power may be challenged and confirmation is weak.

We have two main speculations as to why numbers exist in nature. The first is that the natural number system, that is, the whole positive numbers, which are described by a set of assumptions we call axioms, satisfies as one of these axioms

if n *is a number there exists a* (*unique*) *number* (n + 1).

The claim is that this axiom, which replicates one whole number from another, is and has been physically implemented in some way in the world. So our first speculation is that at some stage or other, and persisting as a consequence to the present day,

the universe has been created as a self-replicating automaton.

We might interpret this as that a state has been chosen, say in an exhaustive selection of possibilities, which has rules with the effect of replicating itself.

We will go further in this speculation, and specify that all iterative processes have been implemented in the universe, not just those of addition. So this includes multiplication (present in the inverse square law of gravity) and exponentiation, which with complex values can represent waves, the most general operation being called a superexponential operation.

Our second speculation concerns another aspect of mathematics, the description of space. We link ideas of space to abstractions derived from the properties of numbers which become descriptions of space. The technical mathematical details of what we are about to describe is available in chapter V on novanions in this eBook, and further applications to physics have also been derived in [Ad17], some of which we describe below.

An interesting aspect of some generalisations of numbers is that for these objects, and I will take as my example 10-novanions, which is also described in chapter V, the 10-novanions have the property that any two 10-novanions may be multiplied together and the result is not zero unless at least one of them is zero, provided the scalar part is nonzero.

Our speculation is that 10-novanions, which have one real component and nine other components, model time for the real component and part of space for the other components, where interaction between these objects in the universe is modelled by trajectories of objects in novanions and interactions as multiplication between them. In this speculation

there is only one place at which novanionic number is not conserved, and this place is at zero time, the beginning of the universe, where novanions created by multiplicative interaction between themselves from zero are possible.

Inside, novanions contain quaternions with one time dimension and 3 space dimensions.

A further aspect of this physics is that we can introduce other objects which are not like quaternions, but two non-zero such different objects can be multiplied together to give zero, so our speculation continues that

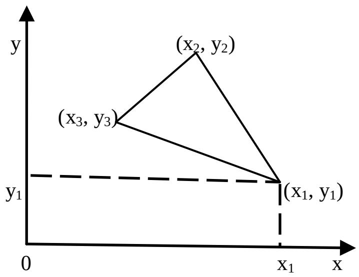
all systems of numbers were initially created, but some systems interacted and self-annihilated, and all that remain are the 'novanion algebras'.

We will introduce in further work on physics the idea that after $t = 0$, the creation time of the universe, it split into a positive time component and a negative time component. Moreover, in the negative time component, provided the space parts, with negative squares, are small compared with the time component, multiplication of two states equates to a positive time component, which registers in the positive universe. So sums of states exist in the negative universe under superposition of states, but multiplication, and therefore interaction, of an even number of states in the negative universe always results in a positive time universe, and multiplication of an odd number of states (which corresponds, say, to the strong three-way interaction of the colour force) keeps its states when in the negative time universe.

What is mathematics?

I think that mathematics consists of three subjects. These are generalisations of number, generalisations of ideas of space, and generalisations of reasoning and logic. It is a realisation that has come to me over the writing of this eBook, that in their full generality all three aspects amount to the same thing. This point of view will be fully developed in Volume III. It is a further speculation that this mathematical structure is the one and only description of the world, which will be investigated in *Universal physics* [Ad17].

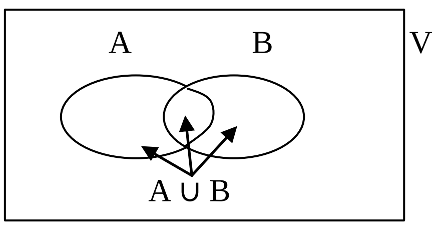
In practice and under current understanding, all three subjects may be put under the first heading, or any other of these. For an example going back to Descartes, we can represent geometry in the plane by two sets of numbers



Each vertex of the triangle above can be represented by its component along the x and y axes. Thus the vertex at (x_1, y_1) may be represented by a pair of numbers, firstly x_1, belonging to the set of numbers given by x, then y_1, which belongs to the set of numbers given by y. If the axes x and y are at right angles, there is only one way that this representation can be given.

Subspaces of this type may be obtained as subsets of the whole space. There is then a connection to a type of logic called propositional calculus. There is a bijection (a two-way mapping) between the algebra of sets and this logic. For example the logic operation OR is related to the set operation ∪ (union), acting on sets A and B, or alternatively on statements A′ and B′; in a model the set A ∪ B is mapped bijectively to the statement ‘x belongs to A’ OR ‘x belongs to B’, denoted

$(x \in A)$ OR $(x \in B)$.



The set V in the above ‘Venn diagram’ is the universe to which A and B belong.

We can introduce the idea of *closed* and *open* sets, and from this develop a model of intuitionalistic sets. A model of a closed interval of a real line

$t \leq x \leq u$

t u

is a segement with the end points t and u included.

A model of an open interval

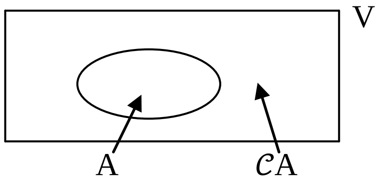
$t < x < u$

is a segment with the boundary points t and u excluded.

We can develop a model of intuitionalistic logic, where for a statement A' we do *not* include the rule

$\mathrm{NOT}(\mathrm{NOT}\ A') \equiv A'$.

A model in sets of NOT is the complement $\mathcal{C}$A of A, the set with elements which do not belong to A. If A is open, then $\mathcal{C}$A is closed, and if A is closed, then $\mathcal{C}$A is open. This can be depicted in the Venn diagram



The intuitionalistic algebra is obtained by always forming the open set of sets like A, which can originally be open, closed or a combination. Then for a bijective mapping, denoted by $\leftrightarrow$,

$[\mathrm{open}\ \mathcal{C}[\mathrm{open}\ \mathcal{C}A]] \leftrightarrow \mathrm{NOT}(\mathrm{NOT}\ A')$. (1)

In the intuitionalistic logic the universe V is usually thought of as being an open set. It is clear that the original set theory can be obtained from the intuitionalistic set theory operating on open sets, by adjoining part of the boundary of sets to form, say, partly open and partly closed sets. This is known as Giraud's theorem.

For a binary logic, there are only two possibilities for an object x, it does exist, denoted by $\exists x$ or it cannot exist, NOT$\exists x$. The model is that a mapping of a set $\{x\}$ of objects x ranges over the objects, and $\exists$ or NOT$\exists$ is given to each x. It is usual, but not necessary, to put all x 'not existing' into an equivalence class, the empty set $\emptyset$.

For an n-ary logic, there are n possibilities for an object x. So its type of existence splits into n types. It is clear that a set in an n-ary logic, even when n is an object that is not a number in the usual sense, can be obtained by adjoining to each element x a value in the n-ary logic. When n is 2, then we could have the situation of $\exists x$ or NOT$\exists x$. Thus when n is greater than 1, such n-ary logics may contain $\exists x$ and NOT$\exists x$, and be included in the elementary theory.

The superexponential algebra is obtained from repeated operations on objects; the first is addition, the second is multiplication and the third exponentiation (denoted sometimes by $\uparrow$), and we can continue this process indefinitely. A feature of these operations, is that after the multiplication type, two of the rules of composition for objects fail at the same time, those of commutativity, since in general

$a \uparrow b \neq b \uparrow a$, (2)

and of associativity, where usually

$(a \uparrow b) \uparrow c \neq (a \uparrow b) \uparrow c$. (3)

This means that an algebra on general objects is not, except possibly for addition and multiplication, given by group theory where for multiplication rather than $\uparrow$ the inequality (3) does not hold, or where both these operations are present is not given by the theory of rings.

However, we will find it possible to develop a general (but nonstandard) superexponential algebra, and this we claim should be the basis of a generalised algebra. There exist superexponential algebras on general objects x which we will develop in the chapters which follow.

For reasoning, we have sketched the rules for using symbols. If we assume these rules act on states, we may express our philosophy as being

> *all mathematics is about objects derived from the idea of number and the rules for changing them.*

These changes are called transformations. We would like to, and believe we can, go further

> *all transformations are states,*

but the existence of irreversible algorithms shows that not all states can be found by transformations. We might think of this as a link between the irreversibility of algorithms like retrieving the selected x_i's of $f(x_i) = \sum_i x_i$ and the second law of thermodynamics in physics, where the entropy, or negative information, of an isolated system never decreases, and it may be impossible to return with certainty from a mixed state to its originating unmixed state.

'How do we prove and develop these results?' is the subject from chapter I onwards.