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The optimal arrangement of reflectors in solar panel tracker arrays

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by Jim Adams, Graham Ennis, Doly García and Tim Gibbs

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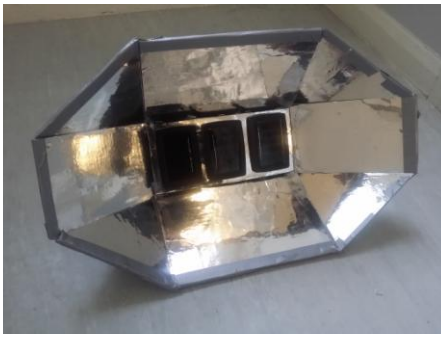
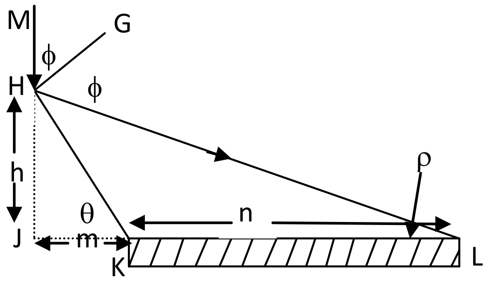
Abstract: *We consider plane mirrors surrounding a solar panel array in which the sun's incident radiation is at right angles to the panel, as it would if the panel were configured to track the sun. A specified constraint is that the incident radiation on the solar panel with reflectors is 3 times that of the PV panel alone, ignoring inefficiencies of the reflecting film. Other magnifications are discussed. We find the optimum configuration of the mirrors is at* 55.7° *to the horizontal axis defined by the* PV (*photovoltaic*) *panel, and the length of this edge reflector is* 0.65 *of the length of the panel, provided the radiation from each reflector covers all of it. We also consider corner mirrors with the same objective. The PV panel considered is square.*

The problem: Consider light shining vertically on a PV panel surrounded by plane mirrors at an angle θ to the horizontal. What is the optimum value of θ so that the entirety of the panel is illuminated?



A PV array with mirrors, not of the optimal design calculated in this paper.

The detailed mathematical solution: We depict the angle, θ radians of the mirror compared with the horizontal panel baseline, the length of the panel, n, the length of the mirror, m/cos θ, and the angle ϕ to the vertical with respect to the mirror with which the sun's rays hit the mirror.



We will assume that at the furthest edge of the mirror a light ray is reflected to hit the far end of the PV panel.

For triangle HKL, the interior angles sum to π radians so, since $G\widehat{H}K$ is a right angle and JKL is a straight line

$$\rho + ((\pi/2) - \phi) + (\pi - \theta) = \pi,$$

or

$$\phi = (\pi/2) + \rho - \theta. \qquad (1)$$

MHJ is a straight line and the interior angles of triangle HJL sum to π radians, so

$$2\phi + ((\pi/2) - \rho) = \pi,$$

or

$$2\phi = (\pi/2) + \rho. \quad (2)$$

Eliminating ϕ from (1) and (2) gives

$$(\pi/2) + \rho = \pi + 2\rho - 2\theta,$$

or

$$\rho = 2\theta - (\pi/2). \quad (3)$$

It is clear from the previous diagram that in this equation $\pi/2 > \rho > 0$.

In triangle HJL

$$h/(n + m) = \tan \rho, \quad (4)$$

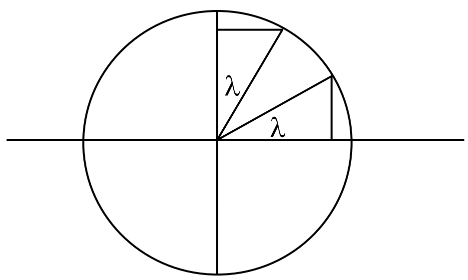
and in triangle HJK

$$h/m = \tan \theta. \quad (5)$$

Thus on eliminating h from (4) and (5)

$$m \tan \theta/(n + m) = \tan (2\theta - (\pi/2)). \quad (6)$$

Now in the diagram



$$\sin((\pi/2) - \lambda) = \cos \lambda = \cos -\lambda$$

$$\cos((\pi/2) - \lambda) = \sin \lambda = -(\sin -\lambda),$$

so putting $\lambda = 2\theta$

$$(n + m) = -m (\tan \theta)(\tan 2\theta). \quad (7)$$

But

$$\sin^2\theta + \cos^2\theta = 1, \quad (8)$$

and

$$\cos 2\theta = \cos^2\theta - \sin^2\theta, \quad (9)$$

so that from (8), (9) gives

$$\cos 2\theta = 2\cos^2\theta - 1 \quad (10)$$

whereas

$$\sin 2\theta = 2 \sin\theta \cos\theta. \quad (11)$$

Thus on putting tan = sin/cos, (7) becomes

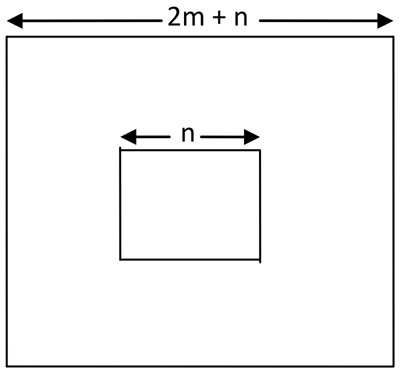
$$(n + m) = -m (\sin \theta/\cos \theta)(2\sin \theta \cos \theta)/(2\cos^2\theta - 1),$$

so

$$[2(n + m) - 2m] \cos^2\theta = -2m + n + m,$$

$$\cos^2\theta = (n - m)/2n. \quad (12)$$

Our specification is that the total output of the PV array with mirrors should not exceed k times the output of the PV array on its own. We are particularly interested in magnification k = 3.



The output of the PV array is 1 unit, so that the output from the mirrors should be (k – 1) units, in order for this to sum to k units. Thus with perfect reflectivity

$$(2m + n)^2 - n^2 = (k - 1)n^2,$$
$$2m + n = (\sqrt{k})n$$
$$m = [(\sqrt{k}) - 1]n/2, \quad (13)$$

so that

$$m = 0.207n \ (k = 2), \text{ or } 0.366n \ (k = 3). \quad (14)$$

Then from (12) in (13)

$$\cos^2\theta = (3 - \sqrt{k})/4$$
$$= 0.39645 \ (k = 2), \text{ or } 0.3170 \ (k = 3)$$
$$\cos\theta = 0.62963 \ (k = 2), \text{ or } 0.5630 \ (k = 3)$$

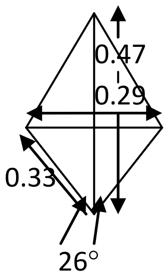
giving

$$\theta = 50.97° \ (k = 2), \text{ or } 55.7° \ (k = 3). \quad (15)$$

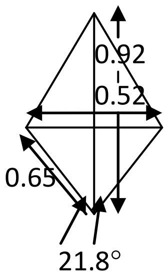
The length of the mirror is $m/\cos\theta$ = 0.33 (k = 2), or 0.65 (k = 3) of the length of the panel.

There is no need to extend the edge reflectors so that they are not rectangles.

If we include 4 mirror diamond shapes at the corners, tipped so that the corner of each mirror meets the corner of the array, then the diagonal for a square array is ($\sqrt{2}$)n, so that if a corner mirror is a diamond shape meeting with edge mirrors of length 0.33n (k = 2) or 0.65 (k = 3) and is again aligned at 51° (k = 2), or 55.7° (k = 3) then given the linear proportionality of the radiation on the array, these 4 corner mirrors spread sunlight over the entire diagonal to the other corner of the PV part of array. However, a calculation shows that a corner mirror, if connected by a diagonal to the edge mirrors, is not at an angle 51° (k = 2) or 55.7° (k = 3) to the horizontal, so the corner mirrors should be set back at the lesser angle of 51°(k = 2) or 55.7° (k = 3). The width of this corner mirror diagonal, which we will assume is approximately the same in the 51°(k = 2) or 55.7° (k = 3) configuration, would be 0.29 units (k = 2) or 0.52 units (k = 3). On the other hand the height of the second diagonal at right angles to this diagonal is of 0.33($\sqrt{2}$) = 0.47 units (k = 2) or 0.65($\sqrt{2}$) = 0.92 units (k = 3) in order for the solar radiation at its tip to meet the other corner of the PV part.



The corner diamond shaped mirror, *magnification* k = 2



The corner diamond shaped mirror, *magnification* k = 3

Note that the diagonal width of a corner mirror does not completely illuminate the PV array. The area of the corner mirror is (1/2)(0.47)(0.29) = 0.07 square units (k = 2), or (1/2)(0.92)(0.52) = 0.24 square units (k = 3) there are 4 of them and they are inclined at an angle 51° (k = 2), or 55.7° (k = 3) to the horizontal, thus the power output from the 4 corner mirrors is $0.07\times4\times0.6296348 \approx 0.17$ of the PV array (k = 2), or $0.24\times4\times0.5630 \approx 0.54$ of the array (k = 3).

We will assume the mirror film has properties similar to 3M™ Cool Mirror Film 330, with an average reflectance at normal incidence of 89% or greater. This reflects light that is useful for PV modules and transmits infrared light. When used in low-x concentrated photovoltaic systems, it can be used to increase the amount of usable light on the module while limiting the amount of heat-generating infrared light on the module. At normal incidence angles, it has low visible reflectance, which limits blinding reflections.

This means that the total configuration can be enhanced to reach $1.0 + 0.89 \approx 1.8$ times the power output of the array alone (k = 2), or 2.8 (k = 3). This is suitable for the maximum tripling capacity of the PV array, also given that the irradiation from the corner mirrors is more concentrated closer to the PV centre.